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Surjective Morphisms of Holomorphic Vector Bundles (Geometric Theory of Several Complex Variables)

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Surjective morphisms of holomorphic vector bundles

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Let E and Q be holomorphic hermitian vector bundles over the complex analytic manifold X , and let g be

$$E \rightarrow Q \rightarrow 0,$$

a surjective morphism of vector bundles over X . We are interested in the fundamental and classical problem to give nice, sufficient conditions about X and E such that the induced morphism

$$g_* : H^0(X, E) \rightarrow H^0(X, Q),$$

(over the global sections) will be also surjective. This is obviously true if, for instance, X is a Stein manifold. In this talk, we shall obtain such a result, using very weak positivity assumptions on X and E .

We suppose X kählerian and weakly pseudoconvex (i.e. there exists a plurisubharmonic exhaustive function of class C^2 over X). Let n be the complex dimension of X . K will denote the canonical line bundle of X . $\det Q$ is the line-bundle qQ (with $q = \text{rang } Q$). Let p be the rang of E . We have the following result:

Theorem. If E is semi-positive in the sense of Nakano (only in the sense of Griffiths if $p - q = 1$), then

$$g_* : H^0(X, E \otimes K \otimes (\det Q)^\ell) \rightarrow H^0(X, Q \otimes K \otimes (\det Q)^\ell)$$

is surjective for all integers $\ell > \inf(n, p - q)$.

The tangent bundle of the projective space P_n , the vector bundle $L \otimes \mathbb{C}^P$ (where L is a semi-positive line bundle) are examples of semi-positive vector bundles in the sense of Nakano.

The following result will be applied to a wide class of vector bundle.

Corollary. If E is the quotient bundle of some vector bundle F of rang N , which is semi-positive in the sense of Nakano, then

$$g_* : H^0(X, E \otimes K \otimes (\det Q)^\ell) \rightarrow H^0(X, Q \otimes K \otimes (\det Q)^\ell)$$

is surjective for all $\ell > \inf(n, N - q)$.

The assumption over E is realized for instance if E is generated by its global sections. For example, the tangent bundle of a homogeneous manifold is generated by its global sections. The corollary is a consequence of the theorem, putting up the section of Q directly in section of F .

Let S be the kernel of g , such that we have the exact sequence:

$$0 \rightarrow S \rightarrow E \rightarrow Q \rightarrow 0.$$

Considering the exact cohomology sequence, the surjectivity of g_* is equivalent to the vanishing of the image of the canonical morphism $\delta : H^1(X, Q) \rightarrow H^1(X, S)$. It is a remarkable feature that we shall obtain the vanishing of the image of δ , but not necessarily the vanishing of all the group $H^1(X, S)$. Therefore our result may be thought as a partial vanishing theorem. Moreover it does not involve a strict positivity assumption, as it is usual in this kind of result.

In the proof we essentially use the fact that the curvature

of the bundles S , E , Q are closely connected with the obstruction of the lifting of sections of Q in sections of E . Using curvature estimates, the Kodaira-Nakano inequality and Hörmander's L^2 -methods, we solve some specific $\bar{\partial}$ -equation with values in S , associated to the morphism δ . Moreover, we obtain some very precise L^2 estimates for the lift of sections of Q , involving the natural parameters of g . These estimates allow to obtain some more general results, concerning morphisms g , which are not everywhere surjective. Even locally these last results are not trivial. We do not reproduce them here for the simplicity. Complete results and proofs can be founded in [3]. Moreover, some extensions of the L^2 Hörmander estimates to non complete kähler and weakly pseudoconvex manifold are proved in [2].

The results of the papers [2] and [3] are in fact extensions of an older paper [1], whose motivations were completely different.

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- [1] Application des techniques L^2 à la théorie des idéaux d'une algèbre de fonctions holomorphes avec poids, Annales Scient. de l'École Normale Supérieure, 5, 1972, 545-579.
- [2] Morphismes surjectifs et fibrés linéaires semi-positifs, Séminaire P. Lelong-H. Skoda (Analyse) 17^e année, 1976-77. Lecture Notes (à paraître).
- [3] Morphismes surjectifs de fibrés vectoriels semi-positifs, préprint, Université Pierre et Marie Curie (Paris VI), 1978.